Conditionals, Part II: Philosophical Logic.

Lecture III, *Stalnaker on Indicative Conditionals*, 24th October

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Let's put to one side the idea that indicative conditionals do not express propositions and look an influential account of their truth conditions—Stalnakerian semantics for conditionals.

1. *Preliminaries*

1.1. Stalnaker's account presupposes the notion of a possible world. If we were only interested in giving a formal account of conditionals, we wouldn't have to worry about what these entities are. But here we are giving an account of what the *truth* of a conditional amounts to, so we should care. Here, we just assume that possible worlds are *total ways the world could have been*: a world w is *total* in that p or $\neg p$ is true at w and *possible* in that w is genuinely a way the world could have been.

1.2. Propositions are defined as functions from worlds to one of two truth values (T — true; F — false), but not both. As such, we simply think of propositions as sets of possible worlds. Each proposition partitions the logical space of possible worlds into those worlds at which it is true and those at which it is false. Two simple examples: if $p = q \wedge \neg q$, then p is the empty set; if $p = q \vee \neg q$, then p is the total set of all possible worlds.

1.3. Stalnaker also appeals to a notion of *similarity* between worlds. We'll talk more about this in coming weeks, but for now think of similarity as inducing, for any world w, a coherent ordering of worlds, staring with w. Similarity will also be context sensitive, so the particular ordering will vary from context to context.

2. *Stalnakerian Semantics for Indicative Conditionals*

2.1. Stalnaker's semantics is motivated by the Ramsey Test, see (Stalnaker, 1968: 32–33). Recall that the idea there was to work out whether you accept 'If p, then q' in cases where you do not accept p, you hypothetically add p to your stock of beliefs, minimally arrange them, and ascertain whether a belief in q is warranted. On Bayesian assumptions about belief, this resulted in The Equation, interpreting probability as degrees of belief. Stalnaker uses *possible worlds* as the 'ontological analogue' of stocks of belief:

2.2. We first define what it is for a conditional to be true relative to a world and then define the truth *simpliciter* as truth at the actual world, @. We define a *selection function*—call it f—from pairs of worlds and propositions. For any world w and proposition p, $f(p, w) = w'$, where w' is the *closest* world at which p is true. That is, the most similar world to w at which p is true. (What if p is true at no worlds, i.e. impossible? Then f selects the absurd world where every contradiction is true.) We then have truth conditions.

(If at w) $p \rightarrow q$ is true at world w iff q is true at $f(p, w)$; false at w otherwise.

(If-true) $p \rightarrow q$ is true iff q is true at $f(p, @)$; and false otherwise.

2.3. How do we assess this theory of conditionals? It won't be particularly helpful to assess natural language conditional claims. "If the butler didn't commit the robbery, the gardener did" is true just in case the closest world at which the butler didn't commit the robbery is a world where the gardener did. But which *is* the closest such world? Instead, we can think about the formal properties of conditionals on this semantics.

3. *Some Features of Conditionals in Stalnakerian Semantics.*

3.1. Many general features of conditionals are well-known. Stalnakerian semantics can account for them. The first feature to look at is the non-transitivity of indicatives.

Non-Transitivity: In general, it does not follow from $p \to q$ and $q \to r$ that $p \to r$.

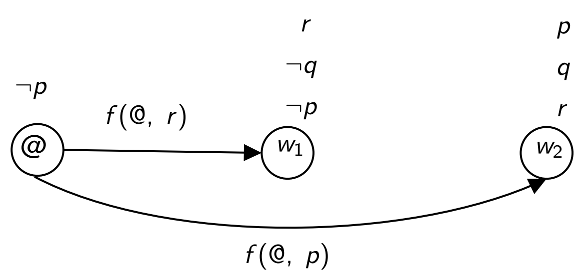
Transitivity holds for the material conditional, i.e., in general if $p \supset q$ and $q \supset r$, then $p \supset r$. Indicatives differ.

(1) If the cows are in the turnip field, then the gate has been left open. (True!)

(2) If the gate has been left open, then the cows have not noticed that the gate is open. (True!)

(3) If the cows are in the turnip field, then the cows have not noticed that the gate is open. (False!)

The truth of (1)–(2) and the falsity of (3) can be explained in Stalnaker's framework. Let $p =$ The cows are in the turnip field, $q =$ Cows notice gate is open, and $r =$ Gate is open. Then consider the diagram:



3.2. On Stalnaker's semantics, contraposition of indicatives fails:

Contraposition: In general, $\neg q \to \neg p$ follows from $p \to q$.

Contraposition fails because for an arbitrary world w, even if q is true at $f(w, p)$, it does not follow that $\neg p$ is true at $f(w, \neg q)$. A good example of this: "If the US halt the bombing, the North Vietnamese will not agree to negotiate" does not entail that "If the North Vietnamese agree to negotiate, then the US will not halt to bombing" (Stalnaker, 1968: 39). Just because the closest world in which the US halts the bombing is one in which the North Vietnamese will not negotiate does not mean that the closest world in which the North Vietnamese agree to negotiate is one in which the US does not halt the bombing.

3.3. Finally, on Stalnaker's semantics, denying a conditional $p \to q$ entails $p \to \neg q$. If $p \to q$ is false, then $f(@, p)$ is not a world at which q. That means $f(@, p)$ is a world at which $\neg q$. Thus, $p \to \neg q$. This is line with how we ordinarily think about conditionals. If you assert "If the freezer is at $-1°$C, then the beer is frozen", I would disagree, asserting: "Beer freezes at $-2°$C. If the freezer is at $-1°$C, then the beer is not frozen".

3.4. Note that not all features of Stalnaker's indicative conditionals seem right. Indicatives with impossible antecedents are universally true which is not in line with judging (4) true, but (5) false.

(4) If $\pi = 3.14$, then the area of a unit circle is 3.14.

(5) If $\pi = 3.14$, then the area of a unit circle is $1,000,000$

This problem runs deep for Stalnaker semantics, since it is thoroughly *intensional* and makes no room for *hypterintensional* distinctions: "$\pi = 3.14$" and "$\pi = 3.15$" express the same necessarily false proposition ($\varnothing$).

4. *Stalnaker on Reasonable Inference, the Direct Argument*.

4.1. The indicative conditional on Stalnaker's semantics is stronger than the material conditional, but weaker than a strict conditional, i.e., $\Box(p \supset q)$. How, then, should we account for the following.

The Direct Argument (DA): Either the butler did it or the gardener did it. Therefore, if the butler didn't do it, the gardener did. ($B \vee G \therefore \neg B \to G$.)

If this is a valid pattern of reasoning, then the indicative conditional is logically equivalent to the material conditional. (Note that $\neg B \vee G$ is logically equivalent to $B \supset G$.)

4.2. The key to Stalnaker's response is the distinction between a reasonable inference and an entailment. The direct argument does not outline a pattern of entailment, but a pattern of reasonable inference. Entailment is defined as expected: if p entails q, then, necessarily, if p is true, q is true. Reasonable inference:

Reasonable Inference: An argument form is reasonable just in case every context in which a premise of that form could appropriately be asserted or explicitly supposed, and in which it is accepted, is a context which entails the proposition expressed by the corresponding conclusion (Stalnaker, 1975: 278).

Stalnaker defines all this in the formal semantics, but we don't have to worry about that here. Here's the thought. (DA) is a reasonable inference because $B \vee G$ is only appropriately accepted in a context in which truly either B or G. By accepting $B \vee G$, all the closest possible worlds to the actual world, in that context, are worlds at which $B \vee G$. Thus, the closest possible world at which $\neg B$ is a world at which G. Thus, $\neg B \to G$.

4.3. (DA) does not encode an entailment, however. To see this note that $B \vee G$ can be true if B. Suppose w is a world at which the butler did it. It doesn't have to be the case that the closest world v to w where the butler didn't do it, is one where the gardener did it. Perhaps it was the chef! So, there's a world at which $\neg(\neg B \to G)$. Thus, it's possible that $B \vee G$ and yet $\neg(\neg B \to G)$ and so there is no entailment.

4.4. Relatedly, the notion of context, appropriate assertion, acceptance, etc., is central to Stalnaker's distinction between indicative and subjunctive conditionals. Asserting an indicative does not commit one to the truth of the antecedent, but it is appropriately asserted if the antecedent is presupposed in the context. The subjunctive mood generally, however, indicates that certain suppositions in the context are being suspended. The closest world, then, will be one *outside* of the context.

References

Stalnaker, Robert (1968). A Theory of Conditionals. In: *Studies in Logical Theory (American Philosophical Quarterly Monographs 2)*. Ed. by Nicholas Rescher. Blackwell, 98–112.

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