The Nature of Logic, Part II: Philosophical Logic.
Lecture I, *The Quinean Challenge*, 16th February.
Christopher J. Masterman (cm789@cam.ac.uk, christophermasterman.com)

1. Some Preliminary Distinctions

1.1. To make any kind of progress on questions about the nature of logic, we should from the outset make some distinctions. We should get clear on what we should mean by *logic* and what kinds of questions we can, and should, ask about its nature, particularly its supposed empirical nature.

Logic: There are variety of things we might be concerned with when we talk about the nature of *logic*.

(i) **Logical Truths**. Some claims are true ('I am cool'), some claims are *logically* true ('If I am cool, then I am cool'). Questions about logic are often questions about certain sentences, logical truths, i.e., questions about that special class of truths—*which* claims are logically true? and *why*?

(ii) **Logical Inference**. Some claims logically following from other claims. Questions about logic are often questions about the special logical relationship(s) which hold(s) between sentences. That is, we can ask questions about the nature of logical *inference*—what is the nature of logical inference? how does it differ from weaker notions, e.g., reasonable inference? is there a unique notion of logical inference? is it an objective notion?

(iii) **Logical Necessity**. One way of understanding (i) is that certain claims hold *in virtue of* logic alone, or they *must* be the case, in some sense. This introduces a notion of logical necessity, distinct from other kinds of necessity, e.g., metaphysics, epistemic, nomic, and so on. Similarly, (ii) could be understood as saying that some claims follow from others as a matter of logical necessity. We might then ask questions about *that* notion—how broad or narrow is it? is it objective?

1.2. Two further points about *logic*. First, we should distinguish between logic or logics in the sense of formal systems which are really only of mathematical interest and logic *proper*—Logic with a capital *L*. There are many different logics in the first sense. Some of the questions above remain interesting if we think of logics as purely mathematical objects, but others do not, e.g., is there a unique notion of logical inference? Second, we should be careful to keep (i) and (ii) apart. Sometimes, the logical truths and inferences are nicely related:

$$(DT) \quad \phi_1, ..., \phi_n \vDash \psi \text{ if and only if } \vDash (\phi_1 \land ... \land \phi_n) \to \psi$$

In which case, we can interchangeably talk about the *inferences* and logical *truths* which capture those inferences in conditional form: the inference from '$\phi_1 ... \phi_n$' to 'ψ' is logical just in case 'If $\phi_1 ... \phi_n$, then ψ' is a logical truth. But, a claim like (DT) fails for many candidates for $\vDash$, e.g., second-order logic is not compact. We should not assume from the outset that *logic proper* is devoid of essentially infinitary inference.

1.3. That's what we might mean by *logic*. What could we ask when we ask: what is the *nature* of logic?

Nature: Shapiro (2000) nicely distinguishes two questions or disputes about the nature of logic:

(i) **Status**. We can ask questions about *what it is that makes* some claim a logical truth, or some inference logical, and so on. We can ask questions about the epistemic and semantic properties of logic—are logical truths knowable *a priori*? are logical truths and inferences *analytic*?

(ii) **Extent**. Even if we agree *generally* on what makes a truth or inference a logical one, we can in principle disagree about *which* truths and inferences are logical. That is, beyond answering questions about logic's status, we then need to settle the extent of logic.

Our answers to (i) affect our answers to (ii) and *vice versa*. But these are separate questions.

2. The Analytic View vs. Empirical View

2.1. One view is that *logic is analytic*. More precisely, the thought is that logical truths are true solely in virtue of meaning and what *that* means is that logical truths are true solely in virtue of the meaning of the logical vocabulary, e.g., $p \supset p$ is logically true because it is true in virtue of the meaning of $\supset$. What about logical inferences? The thought would be that logic inferences are analytic insofar as, if ϕ logically follows from ψ, then ψ must hold if ϕ as determined solely by what the logical vocabulary in ϕ and ψ mean. Where does that leave logical necessity? It is tempted to just identify logical necessity and analyticity, so 'It is logically necessary that ϕ' is equivalent to 'It is analytic that ϕ'. But this can't be quite right: it is analytic that all bachelors are unmarried, it is not plausibly logically necessarily so.

2.2. Now, if logic is analytic, then it is plausibly knowable *a priori*, if knowable at all. Provided we know what the logical vocabulary means, then we can know a logical truth without consulting experience in any important way. Likewise, for our knowledge of logical inferences. Crucially, the claim that logic is *a priori* should not be conflated with the claim that our claims to knowledge of logic are infallible—there *a priori* status concerns the source of our knowledge for them, it does not remove the possibility of error or misjudgement. (Similarly, the claim that logic is *analytic* should not be conflated with the claim that it is rationally impermissible to reject any logical truth, or deny a logical inference. This would only be the case if facts about what our language means were transparent to us; but there are reasons to doubt that.)

2.3. In contrast, many have claimed that *logic is empirical*. But what does this mean? As I will understand it here, this claim concerns the nature of the justification of logic, i.e., the claim is, loosely, about the kind of evidence that is relevant to justify some claim being a logical truth, or some inference being a logical one. The claim is *not* that logic is empirical because it describes *how* we reason, nor is it the claim that logic is 'about' empirical matters, whatever that might mean. At the very least, then, the claim that logic is empirical undermines the claim that it is *analytic* and *a priori*, since on this view logical truths are not just true solely in virtue of meaning—how the world stands is (somehow) importantly relevant—and knowledge of truths as logical in part involve consulting experience.

2.4. Much of what is interesting about this claim that logic is empirical will become clearer over the course of the lectures, as we look in detail at sophisticated attempts to undermine the analyticity of logic in this way. But for the rest of today, I want to look at one *extreme* way in which one might argue that logic is empirical. This is Quine's influential holistic epistemology and his discussion of the status of logic.

3. Quine and Holism

3.1. Let's start with a metaphor which will probably be familiar. For Quine, the totality of our beliefs at a certain time should be thought to have the structure of a web. This is the *web of belief*. Each *node* in the web is a belief, and each node is connected to innumerably many other nodes. The connections between nodes represent how the different beliefs hang together. Experience of the world impinges on the web at the periphery. So, at the periphery, there will be beliefs like 'There is a red patch in my visual field', and so on. Deeper into the web, one finds beliefs on which experience would seem to have less of a bearing, e.g., '2 + 2 = 4'. But crucially, even deep beliefs hang together with beliefs at the periphery.

3.2. For Quine, there is one goal in inquiry and that is to organise one's web of beliefs in order to account for experience in a way which facilitates accurate predictions. Distinctively, Quine holds that in organising one's web in response to experience *any* belief in the web can be revised. (Note that this does not follow simply

from the idea that our beliefs form this web structure.) Of course, Quine notes that more central beliefs to the web will be more difficult to revise, and will require more evidence to revise, but there is no principled barrier to any given belief being revised. What's important presently, is that this includes the parts of an agent's web of belief which represent what is accepted by the agent about logic. What seems to emerge, then, from this way of representing our beliefs and inquiry is that experience (empirical content) is epistemically relevant to all that we accept about logic. This is an extreme rejection of the *a priori* view of logic: *all* logic is empirical!

4, Worries about Quine

4.1. This is an evocative metaphor. One should of course get the details clear on what ways our beliefs *are* like the metaphorical web and on what reasons we have for thinking that our beliefs are like *that*. But we'll ignore those questions. The more important question for us: *where* is logic in the web?

4.2. One idea—Quine's idea—is that logic, or what one accepts about logic, is represented by particular individual *nodes* in the web of belief. This is consonant with Quine's own conception of what logic is—'the systematic study of logical *truths*' (Quine, 1970). But we were careful at the beginning to distinguish between logical truths and logical inferences. In (Quine, 1951), Quine tells us that logical laws, or inference rules, are understood as being further nodes in the web. Shapiro (2000) notes two problems with this idea

(1) Prejudged Compactness: As we noted at the beginning, we can only maintain a reduction of logical inferences to logical truths, if our logic is compact. Thus, built into the structure of the web is the impermissibility of infinitary inferences, i.e., some $\phi_1, ..., \phi_n, ... \vDash \psi$, but no $(\phi_1 \wedge ... \wedge \phi_n \wedge ...) \rightarrow \psi$.

* Shapiro concludes that on this way of representing logic in the web, there is some *a priori* knowledge of logical after all—no inference is both logical and infinitary.
* Whether this objection is successful is not obvious to me. Compactness fails if there are inferences in a logic which do not correspond to finite inferences—(DT) then fails because there just is no relevant conditional sentence. If the language is finite, any essentially infinite inference will give a counterexample. But why should we limit the content of belief nodes to things expressible in finite languages? We *seem* to comprehend infinitary inferences, so why can't that comprehension be represented in the content of the nodes?

(2) Nodes Alone Won't Do: If all logical laws were represented by nodes, how could we understand the connections between nodes. Having $node_1$ be '$\forall x(\phi(x)) \rightarrow \phi(a)$' and $node_2$ be '$\forall x \phi(x)$' would not *alone* have an impact on the web, unless there was some *rule* that required the two nodes to interact in some way. Note, this *rule* can't be another node like $node_3$ as $((\forall x(\phi(x)) \rightarrow \phi(a)) \wedge \forall x \phi(x)) \rightarrow \phi(a)$ because the same problem arises again: we still need a rule for how $node_1$, $node_2$, and $node_3$ interact.

4.3. The immediate lesson to draw from this is that there must be some connections between nodes which are not themselves represented by nodes. Some of those connections will be logical connections. For example, if a node represents P and a node represents Q, there is a node representing $P \wedge Q$. But is this need for connections, and not just nodes, really that threatening to the Quinean idea that *all* logic is revisable? Shapiro (2000: 338) think so. He notes:

> Suppose someone is considering a change in logic, because less drastic measures are not working. Presumably the troubled theorist would follow the model for any change in the web. He would replace the old logic with the new one and see how it comes out. That is, the theorist would examine the consequences of the change in logic for the proposed new web of belief. Consequences? Which logic do we use to assess the consequences of different logics? is there a correct logic for that, and is this super-logic also just a bunch of nodes in the current web? Regress threatens. Is the super-logic analytic, *a priori*, or incorrigible?

That is, to assess the *consequences* of a change in logic, one must have as a fixed backdrop some minimal logic with which one assesses the consequences. It should be stressed that this logic will be *minimal*, but it will be logic nonetheless. If it forms a fixed backdrop in the web, then this minimal logic will be *a priori*.

4.4. I don't think what Shapiro says is obviously true. One response is to think that we must always hold some minimal logic *M* fixed, when deriving the consequences of changing our logic *L* to *L′*, but that the logic *M* is also revisable—just not revisable simultaneously with revising logic *L* to *L′*. In other words, we can't revise *all* our logic *all* at once. But the Quinean view was never committed to such a claim: the claim was only that all logic can be revised, just like anything else.

4.5. For now, we have mostly neglected to talk about logical necessity and how this might figure in arguments for all of logic not being *a priori* and instead revisable in the light of empirical evidence. Focusing on this notion becomes relevant when we consider the kinds of claims made by those who think empirical evidence does have a bearing on our choice of logic. After all, their claim is that logic is revis*able*, and this is a modal notion. Consider, for instance, Bostock. In his discussion of quantum logic, he writes:

> Here too what is needed is just the demonstration that there are 'alternative logics', and that **there is the possibility of a situation** in which a 'total theory' that incorporates one should be, by the same criteria, better overall than its more conservative rival. (Bostock, 1990: 574)

4.6. A crucial question here is what is the operative notion of *possibility*? Standardly, logical possibility is considered a rather broad—perhaps the broadest—notion of possibility. That is, for instance, if ϕ is metaphysically, or physically, or epistemically possible, then ϕ is logically possible. We, broadly, say that ϕ is logically possible if ϕ is true under any reinterpretation of the non-logical vocabulary. Of course, crucially, ϕ being logically possible depends on what is logically true, i.e., what counts as logic. Shapiro worries:

> Is it a *logical* possibility we envision? If so, the only logic we have at our disposal is our own. From that perspective, the paraconsistent logician contradicts himself, and so what he came up with is not a logical possibility. From a classical perspective, the intuitionist and relevance logicians fail to acknowledge some logical truths and valid inferences. The classical logician simply cannot say that it is logically possible for double-negation elimination to be invalid—not while remaining a classical logician. (Shapiro, 2000: 349)

Of course, the revisionist can claim that we could have adopted a different logic in that our *words* could have been governed by different rules. However, this is just not an interesting claim—no advocate of the view that logic is analytic would deny *that*, see (Shapiro, 2000: 349).

4.7. Where does this leave us? None of these objections were decisive. At best, they show that it is difficult to maintain that all logical claims are revisable. Thus, at best, it seems that some logical truths, or basic inferences, could be most plausibly maintained as *a priori*, but not all. This still leaves a lot of interesting questions about the extent of the influence of empirical matters on our choice of logic open.

References

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