The Nature of Logic, Part II: Philosophical Logic.

Lecture III, *Dummett on Putnam on Quantum Logic*, 23rd February.

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Last week, we discussed Putnam's influential proposal that we should revise classical logic in light of empirical arguments—that classical logic should be revised to quantum logic. We spent some time outlining what quantum logic is and looked at two of Putnam's arguments for adopting quantum logic: one concerned quantum complementarity and the other concerned how we should think about the double-slit experiment.

This week, we'll look at some of what could be said against Putnam's proposal, primarily focusing on Michael Dummett's influential assessment of Putnam's argument for quantum logic in (Dummett, 1978).

1. Putnam's Argument Revisited

1.1. Putnam argues for two core claims in (Putnam, 1979).

(i) In light of certain puzzling results in quantum mechanics, we should think of our reasoning about quantum mechanics as done using quantum logic, not classical logic. Such puzzling results include consequences of the uncertainty principle and interpreting the double-slit experiment. The choice is either to revise our logic or adopt what Putnam thinks are bad positions in the philosophy of science.

(ii) This move to using quantum logic in reasoning about quantum mechanics is *not local*. Putnam thinks that we should wholesale abandon classical logic. It is not that quantum logic makes our reasoning about quantum mechanics convenient or that quantum logic is suited for a particular subject matter. Rather, we should *change our logic* from classical logic to quantum logic.

1.2. First, let's focus on (i). Of course, whether we should accept (i) is going to hinge on two claims. First, the claim that it really is a choice between quantum logic on the one hand and bad positions in the philosophy of science. Second, the claim that those bad positions in the philosophy of science *really are* bad. Both of these claims are controversial, see (Maudlin, 2005). We won't discuss any of the positions that Putnam decries as bad philosophy of science in detail but we can start by looking at an initial tension in Putnam's position between his rejection of distribution and the overarching commitment to realism.

2. Complementarity Argument in More Detail

2.1. Recall that a key argument that Putnam gives for preferring quantum logic over classical logic is how it handles quantum complementarity—the consequence of the uncertainty principle that we cannot specify both the precise momentum and position of any given particle. Here is the thought in brief. Classical logic, unlike quantum logic, holds that conjunction distributes over disjunction:

$$A \wedge (B_1 \vee ... \vee B_n) \vDash (A \wedge B_1) \vee ... \vee (A \wedge B_n) \qquad \text{(Distribution)}$$

This, Putnam argues, is a problem. If A is some specific measurement of momentum and $B_1 \vee ... \vee B_n$ is a disjunction of all the possible positions, then the above licenses a disjunction of conjunctions of a specific momentum and position, each of which is false (Putnam, 1979: 179). In contrast, a precise statement of the momentum *alone* A and a disjunction of all possible positions $B_1 \vee ... \vee B_n$ are perfectly legitimate, true statements in quantum mechanics. So, we must give up Distribution and thus classical logic.

2.2. Here's another way of thinking about the same issue Putnam is raising. If each of $A \wedge B_i$ is false and distribution holds, it follows that either A is false or $B_1 \vee ... \vee B_n$ is false. Suppose that A is a true specification of the precise momentum of a particle, it follows, then, if distribution holds that $B_1 \vee ... \vee B_n$ is false. Since $B_1 \vee ... \vee B_n$ is the statement of all possible positions of the particle, we must therefore conclude that the particle has no position *at all*. This is unpalatable for Putnam (1979: 186):

> If I know that S_z [some precise statement of position] is true, then I know that for each T_j [each precise statement of momentum] the conjunction $S_z \wedge T_j$ is false. It is natural to conclude ('smuggling in' classical logic) that $S_z \wedge (T_1 \vee T_2 \vee ... \vee T_R)$ is false, and hence that we must reject $(T_1 \vee T_2 \vee ... \vee T_R)$—i.e. we must say 'the particle has no momentum'. Then one measures momentum, and one gets a momentum—say, one finds that T_M. Clearly, the particle now has a momentum—so the measurement must have 'brought it into being'. However, the error was in passing from the falsity of $(S_z \wedge T_1) \vee (S_z \wedge T_2) \vee ... \vee (S_z \wedge T_R)$ to the falsity of $S_z \wedge (T_1 \vee T_2 \vee ... \vee T_R)$. This latter statement is true (assuming S_z) ... It is as simple as that.

Any particle has *some* momentum and *some* position, even if there is no true complete description of its precise momentum and precise position, see (Putnam, 1979: 184).

2.3. Note that this final point is not simply a reiteration of Putnam's claim that we either embrace quantum logic or bad philosophy science. Rather, it stems from an overarching and deeper commitment to realism, broadly construed. This comes out most clearly in (Putnam, 1979: 184), where he writes:

> Let $S_1, S_2, ..., S_R$ be all the possible positions of a one-particle system S, and let $T_1, T_2, ..., T_R$ be all the possible momenta. Then:
>
> $$S_1 \vee S_2 \vee ... \vee S_R \tag{1}$$
>
> is a valid statement in quantum logic, and so is:
>
> $$T_1 \vee T_2 \vee ... \vee T_R \tag{2}$$
>
> In words:
>
> Some S_i is a true state-description (1′)
>
> and
>
> Some T_j is a true state-description (2′)

Crucially, (1′) and (2′) are not restatements of (1) and (2) 'in words'. For instance, (1) in words would only be something in the form of a long disjunction stating all the possible positions of the particle. (1′) and (2′) instead say that *some* disjunct in (1) and (2) is the true description of the particle's position and momentum, respectively. What would license *this* restatement of (1) and (2)? In short, a kind of realism: Putnam here assumes that since $S_1 \vee S_2 \vee ... \vee S_R$ is true, this means that one of $S_1 \vee S_2 \vee ... \vee S_R$ is true.

2.4. There's no denying that thinking that $S_1 \vee S_2 \vee ... \vee S_R$ is true means that one of $S_1 \vee S_2 \vee ... \vee S_R$ is true is strikingly plausible. But in the context of Putnam's argument it is problematic. As Dummett (1978: 273) notes, Putnam's argument here depends on assuming that truth *distributes* over disjunction:

$$(B_1 \vee ... \vee B_n) \text{ is true} \vDash B_1 \text{ is true} \vee ... \vee B_n \text{ is true} \tag{T-Distribution}$$

Of course, (T-Distribution) is not strictly speaking an instance of (Distribution)—one involves a predicate expression distributing over disjunction and the other involves conjunction distributing over disjunction. So, Putnam's position is not formally incoherent. Rather, the point here is that it is difficult to see what could motivate a rejection of Distribution which preserved a commitment to T-Distribution. Moreover, without T-Distribution, we would have to accept that $(B_1 \vee ... \vee B_n)$ is true, and yet it doesn't follow that any particular disjunct B_i is true. This puts significant pressure on the idea that adopting quantum logic allows us to avoid problematic philosophy of science, and certainly undermines Putnam's claim that we can be realist about quantum mechanics only by adopting quantum logic.

3. Which logical connectives are we talking about?

3.1. Now let's bring in a discussion of Putnam's second claim—(ii) above. I have throughout talked about disjunction and conjunction, with no qualifications, and used the symbols $\vee$ and $\wedge$ both in the context of talking about classical logic and also quantum logic. A natural question which we should discuss is whether we are talking about the same operations in both classical logic and quantum logic. For Dummett, whether we should think of Putnam's conclusion as 'changing our logic' turns on this:

> What needs to be done, to make out the claim that accepting quantum logic would be rightly described as 'changing our logic', is to argue that the logical constants which appear in quantum logic are the same old constants we have always used. (Dummett, 1978: 271)

As Dummett notes, if Putnam's argument that we should change our logic is successful, then there should be some *proposition* which we previously accepted, but which we now reject; Putnam's argument would not be successful if we could still express what we formerly meant by '$\wedge$' and '$\vee$'—in such a case we would be only rejecting a *sentence* which we now found it convenient to give a different meaning.

3.2. Dummett has a nice analogy for the quantum logic vs. classical logic dispute that makes clear the issue here. Suggesting a radical revision of logic—one which involves replacing some of even the most general criteria for determining validity—is like telling a child who knows that the square of a negative number is positive that there is a square root of -1. Here, Dummett thinks that it is no use to tell the child that such a square root is useful in science. Rather, to explain away the mystery, we must explain that we are making use of an *extended* meaning of the word 'number' (Dummett, 1978: 281). Likewise with the suggestion that we should give up adherence to the distributive law governing conjunction and disjunction.

3.3. Putnam is well aware of the challenge posed by the worry that all his arguments show is that there is an alternative logic, quantum logic, which merely uses the same symbols and names for certain operations like disjunction and conjunction. There are two explicit claims Putnam makes in response to this objection:

(1) Similarities between Classical and Quantum Logic. As we stressed last week, many of the standard inferences governing disjunction and conjunction hold in both classical and quantum logic. Thus, Putnam stresses, that a strong case can be made for the claim that the shift to quantum logic does not involve a change in meaning. Unless, Putnam notes, we can establish that the distributive law is *itself* partially constitutive of the meaning of disjunction and conjunction, see (Putnam, 1979: 189–190).

(2) All the worse for the classical meaning. Putnam recognises that, if one takes the meaning of logical constants like $\wedge$ and $\vee$ to be determined by the valid inferences which involve them, then rejecting distribution would amount to changing the meaning. However, Putnam claims, it does not follow from the fact that classical $\vee$ and quantum $\vee$ are distinct logical constants that we *should* or even *can* use classical $\vee$ in an optimal scientific language (Putnam, 1979: 189):

> ...it may be that having such a connective (and 'closing' under it, i.e. stipulating that for all sentences S_1, S_2 of the language there is to be a sentence $S_1 \vee S_2$) commits one to either changing the laws of physics one accepts (e.g. quantum mechanics), or accepting 'anomalies' of the kind we have discussed.

3.3. Dummett does not think that these claims are sufficient to respond to the worry about meaning change. Dummett, first, distinguishes two possible cases which the dispute between classical and quantum logic may fall into. Here, let C be an advocate of classical and N an advocate of some non-classical logic.

(1) *N* rejects the classical meanings of the logical constants and proposes modified ones;

(2) *N* admits the classical meanings as intelligible but proposes at least equally interesting modified ones.

A good example of case (1) is the dispute between intuitionists and classicists: intuitionists simply reject the classical conception of the logical constants, understood in terms of truth-conditions—intuitionists regard the notion of truth-conditions, independent of proof unintelligible. There can be no worry about there being, from the intuitionist's perspective, both intuitionist as well as classical logical constants.

3.4. Generally, in the case of (2), we have two sets of logical constants, one defined by *N* and another by *C*. *N* does not deny that *C*'s understanding of the logical constants is unintelligible. At best, all that can be concluded by *N* is that we have misinterpreted logical constants as they appear in *sentences* for how *C* understands them and we should, for whatever reason, instead understand them as *N* does. There is no *proposition* which *N* dissents to, which *C* assents to. Crucially, Dummett thinks that Putnam's advocacy for quantum logic is a case of (2) because of Putnam's advocacy of realism.

3.5. Dummett thinks this is most clear in Putnam's so-called *idealised operational* interpretation of quantum logic. In brief, Putnam offers an interpretation of the quantum logical constants in terms of a ortho-complemented lattice of *tests*, see (Putnam, 1979: 195–6) and (Bostock, 1990). Roughly, an elementary proposition counts as true if and only if it would be verified by some test. The truth of non-elementary propositions correspond to specific complex tests. The details are unimportant. Dummett makes two observations:

(A) The very notion of a test as it relates to the physical system is construed in **realist terms**. Putnam writes: 'to every physical property *P* there corresponds a test *T* such that something has *P* just in case it passes *T* (i.e., it *would* pass *T*, if *T* were performed)' (Putnam, 1979: 195). Note, it either passes the test or it doesn't, so it either has *P* or it doesn't, from this definition. That is, as Dummett observes:

> The ... assumption that, for every test (of some suitably restricted kind), there exists a property which is revealed by the test, and which, at any given time, each object either possess or fails to possess, is not an operationalist assumption as such, but a *realist* one. (Dummett, 1978: 277)[Dummett's italics.]

(B) When describing how complex tests relate to complex formulae, Putnam utilises *classical* conjunction and disjunction in the meta-language, see (Putnam, 1979: 195–6) and (Dummett, 1978: 278–80).

Given (A), Putnam cannot deny that the classical connectives are intelligible. That is:

> If the the atomic statements are considered to be ones assigning ... a determinate magnitude to some one of various physical quantities at particular times, and the system is thought of as objectively possessing, for each such physical quantity and at each moment of time, a determinate magnitude, then there can be no possible objection to a classical use of 'and' and 'or' under which '*A* and *B*' holds just in case both *A* and *B* hold, and '*A* or *B*' holds just in case either *A* holds or *B* holds or both. (Dummett, 1978: 285–6)

Moreover, given (B)—the fact that Putnam utilises classical conjunction and disjunction to non-circularly define quantum conjunction and disjunction in this idealised operational way—Putnam cannot deny that classical conjunction and disjunction are intelligible. Indeed, classical conjunction and disjunction are used to define quantum conjunction and disjunction so that the relevant definitions are not circular.

4. Quantum Logic without Realism?

4.1. To summarise, Dummett argues that given Putnam's commitment to realism, he cannot deny that classical conjunction and disjunction are intelligible. Thus, despite his protests in (1), the introduction of quantum logic does not warrant a dismissal of classical logic, or the underlying conception of the classical constants, as unintelligible. As such, Putnam's argument preserves both classical conjunction and disjunction. Putnam has thus not shown that we should reject any substantive logical claim in the face of empirical considerations—Putnam fails to establish the need for an empirically motivated *radical* revision of logic.

4.2. We've been largely discussing the issues which arise from Putnam's attempt to reject classical logic whilst maintaining a realist view of quantum phenomena. Why not ditch realism, then? Well, if we were to do this, much of the force of Putnam's *argument* for quantum logic would be lost. After all, an assumption of realism was central to the argument against the *classical* consequences of puzzling quantum phenomena.

4.3. But there is a more serious worry which Dummett only briefly discusses at the end of his discussion of Putnam. There, Dummett argues that the nature of the dispute between classical logic and non-standard logics, like quantum logic, should not be construed as *empirical*. The heart of the dispute is about how we should understand *the meaning* of the logical constants. Thus, it is a dispute to be settle by a *theory of meaning*. No empirical claim can ever warrant a radical revision of logic. An empirical discovery *might* show us that what we previously thought to be valid cannot be so; but we can only mount an argument for a revision of our understanding of notions like validity—a radical revision—by purely philosophical argument:

> Since, if classical logic is admissible alongside the non-classical one, there can be no question of any *rejection* of a classical law, the crucial thesis, from the present point of view ... the negative one according to which, for statements of the relevant kind, meaning cannot be conceived as given in terms of conditions for the possession of truth-values which attach determinately to statements independently of our knowledge ... [this claim/question] is irreducibly philosophical in character; on which we cannot hope can be answered ... by any discovery in quantum mechanics. (Dummett, 1978: 289)

References

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