Modal Propositional Logic (Proof Theory) Worksheet

Worksheet, return attempts of this to my pigeon hole, 3rd floor, GM by 12:00 on Thursday (9th) and will mark them for the next seminar on Friday 10th.

1. Translate the following arguments into the language of modal propositional logic $\mathcal{L}_{\rho}^{M}$. In each case, make clear how the formulae should be interpreted.

(A) (1) It is possible that John is a doctor.
(2) It is possible that John is a architect.
$\therefore$ (3) It is possible that both John is a doctor and an architect.

(B) (1) Necessarily, if the ball is blue, then the ball is red.
(2) It is impossible for the ball to be red.
$\therefore$ (3) It is impossible for the ball to be blue.

2. (a) Define the modal logic K, stating as precisely as possible the language, axioms and transformation rules of the system. (b) How *many* axioms are in the axiomatic base of K?

3. In all of the systems we have covered, we have the derived transformation rule known as Substitution of Equivalents (Eq). State this derived transformation rule as precisely as possible and show that it holds in K.

4. Prove that the following are true.

(a) $\vdash_k L(p \wedge q) \supset (Lp \wedge Lq)$

(b) $\vdash_k (Lp \wedge Lq) \supset L(p \wedge q)$

(c) $\vdash_k L(p \supset q) \vdash_k \Diamond p \supset \Diamond q$

(d) $\vdash_k (L(p \supset q) \wedge M(q \wedge r)) \supset M(q \wedge r)$

(e) $\vdash_k (Lp \wedge Mq) \supset M(p \wedge q)$

5. Define the modal logic T, stating as precisely as possible the axioms and transformation rules of the system. How does T compare to K? Prove that the following hold.

(a) $\vdash_t (L(p \supset q) \wedge L(p \supset q)) \supset (p \supset q)$

(b) $\vdash_t M(p \supset q) \equiv (Lp \supset Mq)$

(c) $\vdash_t (L(p \supset q) \wedge M(p \wedge r)) \supset M(p \wedge r)$

6. Define the modal logic S4, stating as precisely as possible the axioms and transformation rules of the system. How does S4 compare to T and K? Prove the following.

(a) $\vdash_4 M(Lp \supset Mq) \supset M(p \supset q)$

(b) $\vdash_4 MLp \equiv MLMLp$

7. Define the modal logic S5, stating as precisely as possible the axioms and transformation rules of the system. How does S5 compare to T, K, and S4? Prove the following.

(a) $\vdash_5 L(p \vee Lq) \equiv (Lp \vee Lq)$

(b) $\vdash_5 MLp \equiv LMp$

8. Define the modal logic B, stating as precisely as possible the axioms and transformation rules of the system. How does S5 compare to T, K, and S4? Prove the following.

(a) $\vdash_b (Lp \vee Lq) \equiv L(Lp \vee Lq)$

(b) $\vdash_b L(L(p \equiv q) \supset r) \supset (L(p \equiv q) \supset Lr)$