Week III: Modal Propositional Logic (Semantics)

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[*For feedback, hand in your answers at my pigeon hole on 6th floor of GM by 12pm on Thursday (2nd March). Write your name clearly on anything you submit.*]

1. As precisely as possible, define K-validity. Are the following *wff*s K-valid? If so, outline your argument for this in detail. If not, specify a model in which the *wff* is not valid.

(a) $M(p \supset q) \equiv (Lp \supset Mq)$

(b) $M(p \wedge q) \supset (Mp \wedge Mq)$

(c) $(Mp \wedge Mq) \supset M(p \wedge q)$

(d) $M(p \supset (q \wedge r)) \supset ((Lp \supset Mq) \wedge (Lp \supset Mr))$

2. As precisely as possible, define T-validity and S4-validity. Determine whether the following *wff*s are either T-valid or S4-valid, or neither. Explain in detail your answer, specify a model if necessary. Is there a *wff* which is S4-valid, but not T-valid? Why?

(a) $(Lp \wedge Lq) \supset (p \equiv q)$

(b) $Lp \equiv LLp$

(c) $L(Lp \supset Lq) \vee L(Lq \supset Lp)$

3. As precisely as possible, define B-validity and S5-validity. Determine whether the following *wff*s are either B-valid or S5-valid, or neither. Explain in detail your answer, specify a model if necessary. Is there a *wff* which is S5-valid, but not B-valid? Why?

(a) $L(Mp \supset q) \equiv L(p \supset Lq)$

(b) $MLp \supset LMp$

(c) $(LMLp \wedge LML(q \supset {\sim}p)) \supset Mq$

4. Specify a model $\langle W, R, v\rangle$ such that for some worlds $w, w' \in W$, $\sim Rww'$ and in which all theorems of S5 ($\vdash_5 \alpha$) are valid.

5. How do we prove the K-validity, T-validity, S4-validity, B-validity, and S5-validity Theorems? Sketch a proof.

6. Using the relevant validity theorem, show the following.

(a) $\nvdash_k L(p \supset q) \supset M(p \supset q)$

(b) $\nvdash_4 MLp \supset p$

(c) $\nvdash_5 MLLLLLMMp \supset Lp$

7. Let $\mathfrak{M}^S$ be the canonical model of the normal modal system S. Show that if the *wff* α is valid in $\mathfrak{M}^S$, then $\vdash_s \alpha$. You do not have to prove any lemmas which your proof may rely on, but they must be stated precisely.

8. Show how your answer to (7.) can be extend to prove the following.

(a) If α is K-valid, then $\vdash_k \alpha$

(b) If α is T-valid, then $\vdash_t \alpha$

(c) If α is S4-valid, then $\vdash_4 \alpha$

(d) If α is B-valid, then $\vdash_b \alpha$

(e) If α is S5-valid, then $\vdash_5 \alpha$