Lecture One: Propositional Logic and Basic Logical Notions.

1. State the definition of a valid argument.

2. Determine whether the following arguments are valid or invalid. If valid, determine whether the argument is formally valid and state the form of the argument.

(A.) (1) John is a swimmer.
(2) Either John is not a swimmer or John is lying.
$\therefore$ (3) John is lying.

(B.) (1) London is in Norway.
$\therefore$ (2) Either Bob is cool or Bob is not cool.

(C.) (1) Jane is in Oslo.
(2) If Jane is in Oslo, then Jane is in Norway.
$\therefore$ (3) Jane is in Europe.

(D.) (1) Sally is a cat.
(2) Sally is not a cat.
$\therefore$ (3) Sally is made entirely of cheese.

4. In specifying a language, we must specify two elements. What are they?

5. Which of the following concatenation of symbols are well-formed formulae of $\mathcal{L}_\rho$? If any of the following are not well-formed formulae, explain why.

(a) $(p \supset q) \supset r) \supset (\sim t \supset q)$

(b) $((p \supset q) \wedge (\sim p \supset q)) \supset \sim p$

(c) $(p \supset uu) \vee ((p \supset \sim r) \wedge (p \supset r))$

(d) $p \vee \sim p$

(e) $(p \vee \sim p)$

(f) $\Gamma \vdash p$

6. What is a truth table? Give the distinctive truth table for each of $\sim, \wedge, \vee, \supset, \equiv$.

7. Philosophers and Logicians often make use of fewer primitive connectives. For instance, we can define $p \vee q$ in terms of $\sim$ and $\wedge$, i.e., $p \vee q := \sim(p \wedge q)$. Show how we:

(a) Define conjunction $\wedge$ in terms of disjunction $\vee$ and negation $\sim$.

(b) Define material conditional $\supset$ in terms of disjunction $\vee$ and negation $\sim$.

(c) Define material conditional $\supset$ in terms of conjunction $\wedge$ and negation $\sim$.

(d) Define biconditional $\equiv$ in terms of material conditional $\supset$ and conjunction $\wedge$.

(e) Define biconditional $\equiv$ in terms of conjunction $\wedge$ and negation $\sim$.

(*Clue*: To show that your proposed definition works, use a truth table to show that your definition always has the same truth value of the kind of formula you are trying to define.)

8. Determine whether the following *wff* of PL are either valid, contradictory, or neither by the truth table method.

(a) $(p \wedge q) \supset p$

(b) $p \supset (q \supset (p \wedge q))$

(c) $(p \supset q) \supset ((q \supset (r \supset s)) \supset ((p \wedge r) \supset s))$

(d) $(\sim(\sim p \wedge q) \wedge \sim(\sim q \wedge p)) \wedge \sim(\sim p \vee q)$

(e) $(((p \equiv q) \equiv r) \equiv \sim p) \supset p$

9. Determine whether the following *wff* of PL are either valid, contradictory, or neither using interpretations.

(a) $p \supset (p \vee q)$

(b) $p \equiv \sim\sim p$

(c) $\sim((p \supset q) \supset ((q \supset p) \supset (p \equiv q)))$

10. Could there be a *wff* α of PL which is true on every interpretation v, even though α is false on some lines of its truth table? If so, why so? If not, why not?

11. State the semantic definition of Propositional Logic as carefully as possible.

12. What is an axiom system? What is a theorem?