Seminar IV: Which Modal Logic is 'Right'?

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1. Normal Modal Logic

We have looked at the propositional modal logics K, T, B, S4, and S5.

Syntactically: Language of K is $\mathcal{L}_\rho^M$.[1] The axioms of K:

(K) $L(p \supset q) \supset (Lp \supset Lq)$ is a theorem.

(PC) Any substitution instance of a tautology of PL is a theorem.

The primitive transformation rules of K:

(US) The result of uniformly replacing any variable or variables $p_1, ..., p_n$ in a theorem by any *wff* $\beta_1, ..., \beta_n$ respectively is itself a theorem.

(MP) If α and $\alpha \supset \beta$ are theorems, so is β.

(N) If α is a theorem, so is $L\alpha$.[2]

Language of T, B, S4, and S5 is still $\mathcal{L}_\rho^M$. The primitive transformation rules remain the same. To get T we add the following to the axiomatic base of K.[3]

(T) $Lp \supset p$ is a theorem.

To get B, S4, and S5, we add (T) and, respectively, the following.

(B) $p \supset LMp$ is a theorem.[4]

(S4) $Lp \supset LLp$ is a theorem.[5]

(S5) $Mp \supset LMp$ is a theorem.[6]

Semantically: A model-theoretic semantics uses frames.

(Frame) A *frame* $\mathfrak{F} = \langle W, R \rangle$ is an ordered pair, where W is a non-empty set (of 'worlds') and R is a binary relation on W, i.e., for any members $w, w' \in W$, it is determinate whether Rww' or $\sim Rww'$.

We then define models:

[1] $\mathcal{L}_\rho^M$ is the language of propositional modal logic extended with the sentential operator L, where M is defined $\sim L \sim$.

[2] See Hughes and Cresswell (H&C), 1996, pp. 24–25.

[3] See H&C, pp. 41–42.

[4] See H&C, pp. 62–63.

[5] See H&C, pp. 53–54.

[6] See H&C, pp. 58–59.

(Model) A *model* $\mathfrak{M} = \langle W, R, v\rangle$ is an ordered triple, where $\langle W, R\rangle$ is a frame and v is a valuation function. Note, we say that the model $\langle W, R, v\rangle$ is *based on* the frame $\langle W, R\rangle$ v satisfies, for *wff* α, β and $w \in W$:

($\sim^v$) $v(\sim\alpha, w) = 1$ iff $v(\alpha, w) = 0$; 0 otherwise.

($\wedge^v$) $v(\alpha \wedge \beta, w) = 1$ iff $v(\alpha, w) = 1$ and $v(\beta, w) = 1$; 0 otherwise.

($\vee^v$) $v(\alpha \vee \beta, w) = 1$ iff $v(\alpha, w) = 1$ or $v(\beta, w) = 1$; 0 otherwise.

($\supset^v$) $v(\alpha \supset \beta, w) = 1$ iff $v(\alpha, w) = 0$ or $v(\beta, w) = 1$; 0 otherwise.

($\equiv^v$) $v(\alpha \equiv \beta, w) = 1$ iff $v(\alpha, w) = v(\beta, w)$. 0 otherwise.

(L^v) $v(L\alpha, w) = 1$ iff for every $w' \in W$: $v(\alpha, w') = 1$; 0 otherwise

(M^v) $v(M\alpha, w) = 1$ iff for some $w' \in W$: $v(\alpha, w') = 1$; 0 otherwise.[7]

[7] See H&C, pp. 37–39.

If α is true in a $w \in W$ in model $\mathfrak{M}$ we write $\mathfrak{M}, w \vDash \alpha$. If α is valid in $\mathfrak{M}$ ($\mathfrak{M} \vDash \alpha$), α is true at every $w \in W$ in $\mathfrak{M}$. If α is valid in a frame $\mathfrak{F}$, then α is valid in every model based on $\mathfrak{F}$.

(K-validity) Let a *wff* α be K-valid iff α is valid in all frames $\mathfrak{F}$.

(T-validity) ... T-valid iff α is valid in all reflexive frames.

(S4-validity) ... S4-valid iff α is valid in all reflexive and transitive frames.

(B-validity) ... B-valid iff α is valid in all reflexive and symmetric frames.

(S5-validity) ... S5-valid iff α is valid in all equivalence frames.

(X-validity Theorem) If *wff* α is a theorem of X, where X is either K, T, B, S4, or S5, then α is X-valid.[8]

[8] See H&C, pp. 39–41.

(Completeness) If *wff* α is X-valid, where X is either K, T, B, S4, or S5, then α is a theorem of X.[9]

[9] See H&C, Chp. 6.

2. Varieties of Modality and Rightness

'Possibility' and 'Necessity' are vague. We could mean all sorts: metaphysical, physical, nomological, logical, legal, moral, deontic, epistemic etc.

We are concerned with *alethic* modalities: those which concern the way some proposition is true. This includes metaphysical, physical, and logical modality.

Different systems impose different constraints on modality, e.g., if T governs moral necessity, then if you *must* do some action, it is done. (This is false.)

Two notions of 'right' for 'X is the right logic for ...':

Weak Notion: Logic X is the right logic for Y modality just in case all of the theorems of X are true *simpliciter*, when we interpret L and M appropriately.

Strong Notion: Logic X is the right logic for Y modality just in case all of the theorems of X are logically true—required by logic and nothing more—when we interpret L and M appropriately.

3. Dummett's Argument

Kripke claims that it is not even possible that unicorns exist; or at least, being cautious: if there are none, then it is not even possible that there are unicorns. Dummett disagrees. He considers three arguments for the non-possibility of unicorns.

> **Argument One.** Species terms like 'unicorn' behave significantly similarly to singular terms like 'Christopher'. The (pseudo) species term 'unicorn' is introduced in such a way that it is therefore impossible for there to be unicorns.[10]

[10] 'Could there be Unicorns?' pp. 333–334

- Dummett argues that, if this is right, one of two things must have gone on with the introduction of a species term like 'unicorn':
  (a) The term was introduced as part of some deliberate 'romancing'. It is thus the name of a fictional entity.
  (b) The term was introduced using a description. However, that description failed to pick anything out.
- Do either (a) or (b) support the claim that the term 'unicorn' was introduced in such a way that it is impossible for there to be unicorns?
- Dummett thinks: no.[11]

[11] 'Could there be Unicorns?' §§3–4

  * In the case of (a), whilst this may have happened to begin with, it doesn't impose much of a constraint on the term 'unicorn' because it is most plausible that the sense of the term has shifted.[12] From what we know, some people at least did believe unicorns to exist. Thus, the term must have been, at some point before our present usage, elevated beyond a merely fictional term.
  * With (b), Dummett doesn't think that this warrants the conclusion that unicorns cannot possibly exist. The description failed to pick anything out, but it *could have*. Compare the singular term case:

    > Admittedly, in our hypothetical case, it would make no sense to say that *that person*, Charlotte Corday, might or could have existed; but we could properly say that there might have been *such a woman as* Charlotte Corday.[13]

[12] NB. He *does* accept that *if* 'unicorn' was a fictional species term, it would be impossible for unicorns to exist.

[13] 'Could there be Unicorns?' p. 333

> **Argument Two.** This tries to establish the following claim. '... if there are such animals, they are unicorns, provided that all or most of them are of the same species; but, if there are none, there could be no unicorns.[14]

[14] 'Could there be Unicorns?' §6

- Here's what motivates the second part: '... in the absence of any animals we should identify as unicorns if we knew of them, there is no determinate species, actual or hypothetical, to whose members the term 'unicorn' could correctly be applied.'
- In short, what it is to be a unicorn is left too indeterminate and thus, in the absence of actual unicorns, there is no fact of the matter of what it

is to be a unicorn. There are thus no unicorns in any possible world, if there are none in the actual world.

- Dummett thinks that the conclusion of this argument is incoherent. Dummett argues that it is simply incoherent to state the following.

  (*) There are no unicorns and it is impossible for there to be unicorns, but if there were unicorns, it would be possible.

Argument Three.[15] To resolve the incoherence, argument three takes the reasoning of argument two further and '...holds that we neither shall nor should ever be right to call any animals 'unicorns'; more modestly, that, if we ever do, we shall not be using the word in its present sense, no matter how closely those animals resemble the unicorns of the pictures.'

[15] 'Could there be Unicorns?' §7

- Just as there is no fact of the matter for the difference between two species which fit the under-specified criteria for being a unicorn in two different possible worlds, likewise there is no fact of the matter for the difference between two species which fit the under-specified criteria for being a unicorn in two separate parts of the globe.
- Is this enough to resolve the inconsistency in the conclusion of argument three? Dummett thinks not: he already argued that we should in fact say that there are unicorns, if we find creatures which resemble them. Thus, argument three collapses into argument two.
- A further premise is needed:

  > What argument (3) shows ... is that, if there are two unrelated species, both resembling unicorns ... in external appearance, the members of neither can rightly be called 'unicorns' without changing the existing sense of that word. It does not follow that, if there were in fact only one such species, its members could not rightly be called 'unicorns' ... a further premise is needed. This would have to be that if, ... a term like "unicorn" applies to a given object, it must do so independently of the existence or non-existence of anything that played no part in that object's coming into being ... it would never be in the present sense of the word 'unicorn' to apply it to the members of any species if there was so much as a possibility of there being some unrelated species whose members could with equal right have come to be called 'unicorns'.[16]

  [16] 'Could there be Unicorns?' p. 343

  * Dummett rejects this additional premise. This premise doesn't apply to singular terms, so why should it apply to species terms?

So, Dummett thinks that there might have been unicorns. He also accepts that the nature of unicorns is under-determined. He thinks this has the consequence that B is the incorrect logic for metaphysical modality in the weak sense of the 'correct'.

Argument against B. What it is to be a unicorn is indeterminate. In fact, what it is essential to being a unicorn is indeterminate.

> The pictures and verbal descriptions of unicorns do not suffice to determine what their biological affinities would be, if there were any. They might ... be of the order Artiodactyla, like deer, or of the order Perissodactyla, like horses ... there are no unicorns in the actual world w, but there is a possible world u in which there are unicorns, which belong to the order Artiodactyla, and another possible world v in which there are also unicorns, which in that world belong to the order Perissodactyla.[17]

[17] 'Could there be Unicorns?' p. 345

Let U^A = unicorns belong to the order Artiodactyla. Let U^P = unicorns belong to the order Perissodactyla. In u, U^A holds and, plausibly LU^A holds. In v, U^P holds and, plausibly, LU^P holds.

Both u and v are possible relative to the actual world w, but not *vice versa*.

- If Ruw, then, given that LU^A holds at u, U^A holds at w.
- If Rvw, then, given that LU^P holds at v, U^A holds at w.

If both Rvw and Ruw, then both U^A and U^P hold at w. Plausibly, this cannot be case. Thus, w is not accessible to at least one of v and w. Thus, R is not symmetric. Thus, B cannot be correct.[18]

[18] 'Could there be Unicorns?' p. 346

4. Salmon's Argument

Salmon's argument targets S4. It starts with a modal intuition.

Salmon's Intuition '... a particular material artifact ... could have originated from matter slightly different from its actual original matter m^* ... but not from entirely different matter'[19]

[19] 'The Logic of What Might....' p. 5

Salmon's argument against S4 Salmon argues that Woody—a wooden table—actually originated from matter m^* and it couldn't have originated from m, however, it could have originate from m' and, *were* it to, it could originate from m. Thus, it is not true that 'if Woody necessarily does not originate from m, then it is necessary that Woody necessarily does not thus originate.'[20]

[20] Ibid.

Salmon frames this using impossible worlds: ways the world could not be.

- Since Woody cannot have originated from m, it follows that the way the world is such that Woody originates from m, W, is impossible.
- However, Salmon argues that there is a way the world could be W' such that Woody originated from m' and, from the perspective of W', W is not impossible: it is a way the world could have been.

Objection. The standard objection, voiced by David Lewis[21], is that, although some modalities like physical necessity require we restrict accessibility, when it comes to metaphysical modality, 'the hallmark of metaphysical necessity or necessity *tout court* is that it is completely unrestricted.'[22]

[21] "Say I: This is no defence [of the essentialist doctrine that a table could not have originated from entirely different matter], this is capitula- tion [to radical anti-essentialism]. In these questions of haecceitism and essence, by what right do we ignore worlds that are deemed inaccessible? Accessible or not, they're still worlds. We still believe in them. Why don't they count?" (On the Plurality of Worlds, Oxford: Basil Blackwell, 1986, p. 246).

[22] 'The Logic of What Might...' p. 9

Another way of framing this objection: the notion of a 'metaphysically impossible world is incoherent'. The claim: any world is possible in the unrestricted, metaphysical sense. Insofar as Salmon uses the notion of an impossible world, he has lapsed into an egregious form of anti-essentialism: he actually ends up claiming that Woody *could* have originated from entirely different matter.[23]

[23] 'The Logic of What Might...' p. 9–10

Salmon's response. Salmon responds in several ways.

(i) This objection, he claims, 'conflates two notions that should be kept ... distinct: the generic notion of a way for things to be and the peculiarly modal notion of a way things *might have* been.'[24]

[24] 'The Logic of What Might...' p. 11

(ii) Metaphysical necessity is *not* the limiting case, or the unrestricted sense of possibility. Think of logical modality. Also, consider 'mathematical necessity': 'a proposition is mathematically necessary if its truth is required by the laws of mathematics alone'.[25]

[25] 'The Logic of What Might...' p. 13

Salmon then considers what he calls the 'Ostrich Approach':

Ostrich Approach 'One may chose to ignore ways things could not have been, confining one's sights always and without exception to ways things actually might have been. One may stipulate that a proposition is necessary with respect to an arbitrary possible world w if and only if it is true in every world accessible to the actual world ...'[26]

[26] 'The Logic of What Might...' p. 20

For Salmon, the Ostrich Approach is a bad idea.

(i) In 'any standard or conventional sense ... it is simply incorrect to say 'It is necessary that such-and-such' when there is a possible scenario according to which it is possible that it is not the case that such-and-such...'[27]

[27] 'The Logic of What Might...' p. 21

(ii) On the Ostrich Approach, the distinction between the generic and the modal notion of a world loses all significance.

(iii) The Ostrich Approach conflates 'possibility' and 'actual possibility'. These are, however, distinct notions.[28]

[28] 'The Logic of What Might...' p. 23

Salmon then discusses what one might say against the idea that B is the right logic in the strong sense. He also discusses what one might say to argue against S4 being right in the strong sense, even if you reject his arguments against S4 being right in the weak sense.

He writes: 'Believers in B modal logic ... must claim that it is an essential property of the way things actually are that things might have been that way. These claims are versions of essentialism.'[29]

[29] 'The Logic of What Might...' p. 26

Moreover, '... the essentialism espoused must be held to be not merely metaphysically true but true by the very *logic* of necessity and possibility ... This does not weaken the import of the essentialist claims. On the contrary, the

logical nature of the claims makes them extremely strong versions of essentialism. The claim is not merely that such-and-such worlds are essentially thus-and-so, but that they are essentially thus-and-so *by logic alone*.'[30]

[30] 'The Logic of What Might...' p. 27

This kind of consideration extends to an argument against S4 being right in the strong sense, if not the weak sense: 'The difficulty [for S4 modal logic] stems from a widely shared modal intuition, to the effect that some small variations in the origin of a material artifact is possible whereas complete variations is impossible. Even if one does not share this intuition, however, it should be quite obvious that the modal position of one (such as myself) who canonizes the intuition into metaphysical doctrine is at least coherent. The position cannot be summarily dismissed on logical grounds alone, as one would (rightly) dismiss the position of someone who proposes ... rejecting the characteristic principle of T.'[31]

[31] 'The Logic of What Might...' p. 31

Questions.

1. Do you find Dummett's argument against B convincing? If not, why not?
2. What are possible worlds? What are impossible worlds? Do they exist?
3. Do you find Salmon's argument against S4 convincing? If not, why not?