Week V: Simple Quantified Modal Logic

Instructor: Christopher James Masterman (email: c.j.masterman@ifikk.uio.no)

[*For feedback, hand in your answers at my pigeon hole on 6th floor of GM by 12pm on Thursday (9th March). Write your name clearly on anything you submit.*]

1. Specify, in as much detail as possible, the language of Lower Predicate Calculus, $\mathcal{L}_{\forall}$

2. Specify, again in as much detail as possible, the axioms and primitive inference rules of the Lower Predicate Calculus. With this in mind, define the system LPC + *S*, where *S* is a normal modal system.

3. Show the following, where *S* is any normal modal system and α, β are *wff* of $\mathcal{L}_{\forall}$.

$\vdash_{\text{(LPC+S)}} \forall x L(\alpha \supset \beta) \supset L(\exists x \alpha \supset \exists x \beta)$.

4. What is the language of the Modal Lower Predicate Calculus, $\mathcal{L}_{\forall}^{M}$? Specify, in as much detail as possible, a model-theoretic semantics for $\mathcal{L}_{\forall}^{M}$. Then show how we can semantically define the logics LPC + *S*, for the cases where *S* is K, T, S4, B, or S5.

5. Show that none of the following are theorems of LPC + S5, i.e., specify a model in which, at some world, and under some assignment, the following fail to hold:

(a) $\exists x \sim L \phi x \supset M \forall x \sim \phi x$

(b) $(L \exists x \phi x \wedge \forall x M \psi x) \supset M \exists x (\phi x \wedge \psi x)$

(c) $\forall x (L \phi x \vee L \sim \phi x) \vee \forall x (M \phi x \wedge M \sim \phi x)$

6. What is the Barcan Formula (BF)? Does it have problematic instances? Is it also the case that '$L \exists x \phi x \supset \exists x L \phi x$' is a theorem of LPC + *S*, for any normal modal system *S*?