Frege and Russell on Names and Descriptions. Part IA Meaning.
Lecture II, *Frege: Sense and Reference Unleashed*, 27th February.
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Last week, we looked at Frege's sense/reference distinction for names. Here's a quick run-down:

- **Names have sense *and* reference**. The reference is what the name picks out and the sense is the mode of presentation—**a way of thinking about** the referent. E.g., 'Phosphorus' refers to Venus; but its sense is conveyed by 'the brightest celestial body in the morning which is not the Sun or Moon'.

- **Co-referring names can have distinct senses**. Two names can refer to the same object, but they can have different senses. For example, 'Phosphorus' and 'Hesperus' refer to the same thing, Venus, but have different senses: unlike 'Phosphorus', the sense of 'Hesperus' is conveyed by 'the brightest celestrial body *in the evening* which is not the Sun or Moon'.

- **Frege distinguishes between a name's customary reference and indirect reference**. In ordinary contexts, the reference of a name is its **customary reference**: the thing it refers to. So, in 'Phosphorus is bright', 'Phosphorus' refers to Venus. In propositional attitude contexts, however, the reference of a sense is its **indirect reference**. This is just its sense. So, in 'Jim believes that Phosphorus is bright', 'Phosphorus' refers to the indirect reference, i.e., the *sense* of the term 'Phosophorus'.

- **Cognitive attitudes track sense**. Frege holds that if two sentences have the same sense, then we cannot have diverging cognitive attitudes, e.g., believe, think, know, etc., towards those two sentences. We can have diverging cognitive attitudes towards 'Phosphorus is Phosphorus' and 'Phosphorus is Hesperus'. Loosely, Jim thinking that Phosphorus is Hesperus relates Jim to the senses of *both* 'Phosphorus' and 'Hesperus'. Jim thinking that Phosphorus is Phosphorus relates Jim to the sense of 'Phosphorus'.

- **Sense is objective**. Sense is **objective** insofar as it relates to the inter-subjective, shareable aspect of meaning. We can both think the same thought. Frege, then, distinguishes the sense of a name from the private *idea* one might associate with the name, e.g., mental image or feeling.

This week, we'll look at Frege's distinction between sense and reference *more broadly* applied to other kinds of expressions, not just names, and some other problems the distinction is supposed to resolve.

1. Sentences *refer* to truth values.

1.1. Importantly, Frege applies the distinction between sense and reference to more than just names. Of particular interest, is that Frege applies it to *sentences*. Just as names have a reference and a sense, sentences have a reference and a sense. Throughout, we have often talked about the sense of a sentence, e.g., we said that the sense of a name contributes to the sense of the sentences in which it occurs. But we ought to look at this doctrine in more detail. What is the sense of a sentence? What is its reference?

1.2. Frege identifies the reference of a sentence with its **truth value**. So, all true sentences refer to the same thing (what Frege calls **The True**) and all false sentences refer to the same thing (what Frege calls **The False**). Such a thesis is, at first glance, startling. It's helpful to break this thesis down into two.

> **Thesis (1)**. Sentences, as well as names, refer—they have *referents*.

> **Thesis (2)**. Sentences refer to truth values: The True or The False.

Both of these claims are surprising. Since we naturally associate reference with naming, (1) seems committed to the idea that sentences really are, or behave very much like, names for truth values. Moreover, (2)

involves talking about truth values as *objects*. But this doesn't seem in line with how we usually think about them: I say that a sentence *is true* just like I say a ball *is blue*. That is, I *predicate* truth of sentences and it is quite unnatural to think that a true sentence actually refers to a special object 'The True'.

1.3. To understand why Frege accepts (1) and (2), it's easiest to take them in reverse. Suppose we accept that sentences do indeed have a reference. If we accept that they refer, why should we think that they refer to their truth value? First, Frege accepts a key compositionality principle about reference for sentences:

(3) **Compositionality of Reference**: If two expressions co-refer, uniformly replacing one for the other in any given sentence doesn't change the reference of that sentence.

(3) is similar to other principles we looked at last week. But (3) states how the reference of the sentence as a whole is governed by the reference of its parts. Given (3), we can give an argument that the reference of a sentence is its truth value, see (Noonan, 2001: 143) for discussion. Consider the following four sentences.

(4) Kami Rita is the record holder for the most Everest ascents.

(5) Kami Rita is the man who has climbed Everest twenty-eight times.

(6) The number of times Kami Rita has climbed Everest is twenty-eight.

(7) The number of dominoes in a standard domino set is twenty-eight

By compositionality of reference, (4) and (5) have the same reference, since 'the record holder for the most Everest ascents' and 'the man who has climbed Everest twenty-eight times' co-refer to Kami Rita. By compositionality of reference, (6) and (7) have the same reference, since 'the number of times Kami Rita has climbed Everest' and 'the number of dominoes in a standard domino set' co-refer to twenty-eight. Moreover, (5) and (6) plausibly have the same reference, since (6) is a rewording of (5).

1.4. If the move from (4) to (5), from (5) to (6), and from (6) to (7) preserve reference, then (4) and (7) have the same reference. But what then could that reference be? It seems that the important thing in common between (4) and (7) is that (4) and (7) are both *true*. That is, (4) and (7) co-refer to The True. We could run the same argument with different sentences to show that all false sentences co-refer to The False.

2. Frege on functions, function signs, predicates, and concepts.

2.1. Of course, this argument for (2)—that sentences refer to truth values—assumed (3) and, first of all, that sentences had references at all. Why does Frege think that sentences have references? To understand this, we have to look closer at Frege's philosophy of language. We start with what he says about functions.

2.2. In (Frege, 1980), Frege discusses how we should understand mathematical functions. Loosely, these are things like $2x + 1$, $sin(\theta)$, $y^2 + x$, and so on. What is a function? You might think that the function is the *reference* of a certain mathematical expression. But this can't be right. For instance, '$2(2) + 1$' does not refer to a function of 2, but rather the number 5. Generally, these expression denote the *value* of the function.

2.3. Instead, then, you might think that functions are denoted by mathematical expressions where the number is indicated *indefinitely* by a letter, like 'x'. For example: $2x + 1$. Frege thinks this is closer to the truth, but not still not quite right. What do we mean by 'indicated indefinitely'?

(8) Indefinite Numbers: 'Indicated indefinitely' could mean that we somehow invoke 'indefinite numbers'.

(9) Open: 'Indicated indefinitely' could mean that it is *left open which* numeral goes in the place of 'x', and we just indicate that *some* specific numeral goes in the place of 'x'.

But, first, indefinite numbers do not exist. So, (8) can't be right. Second, if all we are doing is leaving open *which* specific numeral goes in the place of x, the referent of '$2x + 1$' will still be a number and not a function—we just leave it open *which* number it is. But then we may as well have just written 'x', not '$2x + 1$'!

2.4. Now, Frege does think that talking of 'x' as indicating indefinitely gets us close to the correct way of understanding functions. Frege's claim is that the sign for a function is the **common element** which is left over when we *remove* the designations of the arguments. That is, whilst '$2(3) + 1$', '$2(4) + 1$', '$2(5) + 1$' all denote specific numbers, the common element in each of these is the **function sign**. We might illustrate this by writing something like '$2() + 1$'. Crucially, what is distinctive about function signs is that they are **unsaturated** or **incomplete**. Likewise with what those function signs pick out—**functions are unsaturated**.

2.5. So, any complex mathematical expression like '$2(2) + 1$' does not refer to a function, but rather refers to the value of the function for that argument—in this case, the number 5. The function sign is instead the common element such expressions share. Now, what bearing does this have on Frege's philosophy of language? Frege's first distinctive move is to note that unlike mathematical functions like $2x + 1$ which take a numerical argument, e.g. 2, and gives a numerical value, e.g., 5, there are also mathematical functions like $x > y$ which take numerical arguments and give, not a numerical value, but a *truth value*:

(10) If $x = 3$ and $y = 1$, then $x > y$ is true.

(11) If $x = 3$ and $y = 4$, then $x > y$ is false.

Thus, since the moral of Frege's discussion of functions above was that '$2(2) + 1$' doesn't denote a function, we should likewise think that '$3 > 1$' does not denote a function. Now, '$2(2) + 1$' denotes the number 5—the value of the function for that argument. So, what does '$3 > 1$' denote, if not the function? Answer: it denotes a truth value! But strikingly, '$3 > 1$' is not a singular term: '$3 > 1$' is a mathematical sentence. (Think: $3 > 1 = 2 > 1$ is not a well-formed claim.) So, from Frege's discussion of mathematical functions, we seem to arrive at the conclusion that, at the very least, some *mathematical* sentences have truth values as referents.

2.6. Frege's second distinctive move is to generalise this idea to *all sentences*, not just mathematical ones, by understanding **predicate expressions** as similarly **incomplete or unsaturated signs**. Frege calls the reference of a one-place predicate a **concept** and this should be understood as a function from an object to a truth value. Again, just as the sign for a mathematical function is the common element or pattern left over, so too is the sign for the concept. For example, part of 'Jim is tall' refers to a concept. This is a function which takes objects and returns a truth value, depending on whether that object is tall or not.

2.7. Just as '$3 > 1$' did not refer to a function, a sentence like 'Jim is tall' does not refer to any concept. Rather, the reference of 'Jim is tall' is a truth value. To stress: concepts are very different from objects. Concepts are unsaturated or incomplete entities, objects are not. In fact, Frege's conception of object is very general: an object is anything which is not a function. The truth values are objects in this sense.

3. The Sense of Sentences

3.1. The reference of a sentence is its truth value. What then of its sense? For Frege, the sense of a sentence is what he calls **the thought**. We should be careful, again, to keep Frege's notion of thought away from psychology. Rather, the thought is the sense of a sentence, so it is the public, shareable, and objective *content* of a sentence. If we both think *that* Superman is cool, we both grasp the very same thought.

3.2. The need to distinguish the reference from the sense of a sentence arises for very similar reasons which prompted us to distinguish between the sense and the reference of a name. That is, according to the idea

that the reference of a sentence is its truth-value, *any* two true sentence co-refer, e.g., 'Paris is the capital of France' and '$Sin(180°) = 0$' co-refer. The worry is that this leaves us unable to distinguish true sentences which express completely different claims. The solution is that two true sentences can co-refer and yet have distinct senses, thus expressing different thoughts, just as two co-referring names can have distinct senses.

3.3. Frege similarly distinguishes between the **indirect reference** and **customary reference** of a sentence, just as there is a distinction between the indirect reference and customary reference of a name. The customary reference of a sentence is its **truth value**. The indirect reference of a sentence is its **sense**. In propositional attitude contexts, the reference of a sentence is its sense. Why? Consider:

(12) Lois thinks that Nicolas Cage is weird

An initial problem for us now is that, since any two true sentences co-refer, by the compositionality of reference, we should be able to replace 'Nicolas Cage is weird' in (12) for 'Nicolas Kim Coppola is weird' and, again, get the (at least potentially) false 'Lois thinks that Nicolas Kim Coppola is weird'. For Frege, however, the reference of 'Nicolas Cage is weird' in (12) is not its customary reference, but its sense. The sense of 'Nicolas Cage is weird' is distinct from the sense of 'Nicolas Kim Coppola is weird'.

3.4. Now that we have clarified some important elements of Frege's philosophy of language, we can discuss a final distinctive consequence of Frege's thinking about names—his treatment of non-denoting singular terms. Consider the name 'Harry Potter'. This doesn't denote anything. What, then, should we say about:

(13) Harry Potter has a scar on his head

Is (13) true or false? A natural thought is that it is false *because* there is no Harry Potter. But, then, what about 'Harry Potter doesn't have a scar on his head'? This seems like the negation—should this be true?

3.5. Frege thinks that sentences which contain non-denoting terms like 'Harry Potter' are **neither true nor false**. That is, such sentences are non-denoting because the terms in them are non-denoting. Given Frege's functional understanding of predicates, a sentence like (13) which combines a singular term 'Harry Potter' and a concept sign '... has a scar on his head' denotes The True if the referent of '... has a scar on his head' has the value The True when given the referent of 'Harry Potter' as argument. But it doesn't *because* there is no referent of 'Harry Potter'. Likewise, (13) cannot denote The False.

3.6. Crucially, however, although such sentences do not have a reference, i.e., are neither true nor false, they nonetheless have a sense. That is, (13) expresses a thought. This is because Frege allows that even non-denoting names like 'Harry Potter' nonetheless can have a sense. This is a nice feature of Frege's theory, since it allows us to preserve what appears to be the meaningfulness of (13). Moreover, we can distinguish the sense of (13) from the sense of 'Ron Weasley has a scar on his head'. They express different thoughts. In summary, sentences which contain non-denoting terms are neither true nor false but they are nonetheless meaningful because we should distinguish the sense and the reference of a sentence.

References

Frege, Gottlob (1980). Function and Concept. In: *Philosophical Writings of Gottlob Frege*. Ed. by Peter Geach and Max Black. Blackwell.
Noonan, Harold W. (2001). *Frege: A Critical Introduction*. Malden, MA: Polity.